Example ① Find the area between

$y = x^3$ and $y = x - 5x^2$

② Find

$$\int \frac{\cos(\sqrt{b})}{\sqrt{b}}\, db.$$

③ Find $\frac{d}{dx}\left(\int_x^2 p^2 e^{p^2}\, dp\right)$

$$\frac{d}{dx}\left(\int_x^2 p^2 e^{p^2}\, dp\right) = \frac{d}{dx}\left(-\int_2^x p^2 e^{p^2}\, dp\right)$$

$$= -\frac{d}{dx}\left(\int_2^x p^2 e^{p^2}\, dp\right)$$

$$= \boxed{-x^2 e^{x^2}}$$

④ Find $F'(x)$ if

$$F(x) = \int_x^{x^2} \cos(\theta^2)\, d\theta$$

Hint: $\int_a^b g(x)\,dx + \int_b^c g(x)\,dx = \int_a^c g(x)\,dx$

$$\Rightarrow F(x) = \int_x^1 \cos(\theta^2)\,d\theta + \int_1^{x^2} \cos(\theta^2)\,d\theta$$

$$\Rightarrow F'(x) = \left(\int_x^1 \cos(\theta^2)\,d\theta\right)' + \left(\int_1^{x^2} \cos(\theta^2)\,d\theta\right)'$$

$$= \left(-\int_1^x \cos(\theta^2)\,d\theta\right)' + \left(\int_1^{x^2} \cos(\theta^2)\,d\theta\right)'$$

$$= -\cos(x^2) + \cos((x^2)^2)\cdot(x^2)'$$

$$= \boxed{-\cos(x^2) + 2x\cos(x^4)}$$

② Find

$$\int \frac{\cos(\sqrt{b})}{\sqrt{b}}\,db$$

let $u = \sqrt{b} = b^{1/2}$

$du = \frac{1}{2}b^{-1/2}\,db = \frac{1}{2\sqrt{b}}\,db$

$\Rightarrow 2du = \frac{1}{\sqrt{b}}\,db$

$$\Rightarrow \int \frac{\cos(\sqrt{b})}{\sqrt{b}}\,db = \int \cos(u)\cdot 2du = 2\int \cos(u)\,du$$

$$= 2\sin(u) + C = \boxed{2\sin(\sqrt{b}) + C}$$

Check: $\left[2\sin(\sqrt{b})\right]' = 2\left[\sin(\sqrt{b})\right]'$

$= 2\cos(\sqrt{b})\cdot(b^{1/2})'$

$= \cancel{2}\cos(\sqrt{b})\cancel{\tfrac{1}{2}}b^{-1/2} = \dfrac{\cos(\sqrt{b})}{\sqrt{b}}$. ✓

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① Find the area between

$y = x^3$ and $y = x - 5x^2$

<u>Intersection pts</u>: $x^3 = x - 5x^2$

$\Rightarrow x^3 + 5x^2 - x = 0$

$x(x^2 + 5x - 1) = 0$

$\Rightarrow x = 0$ or $x^2 + 5x - 1 = 0$

$(x \quad)(x \quad) = 0$

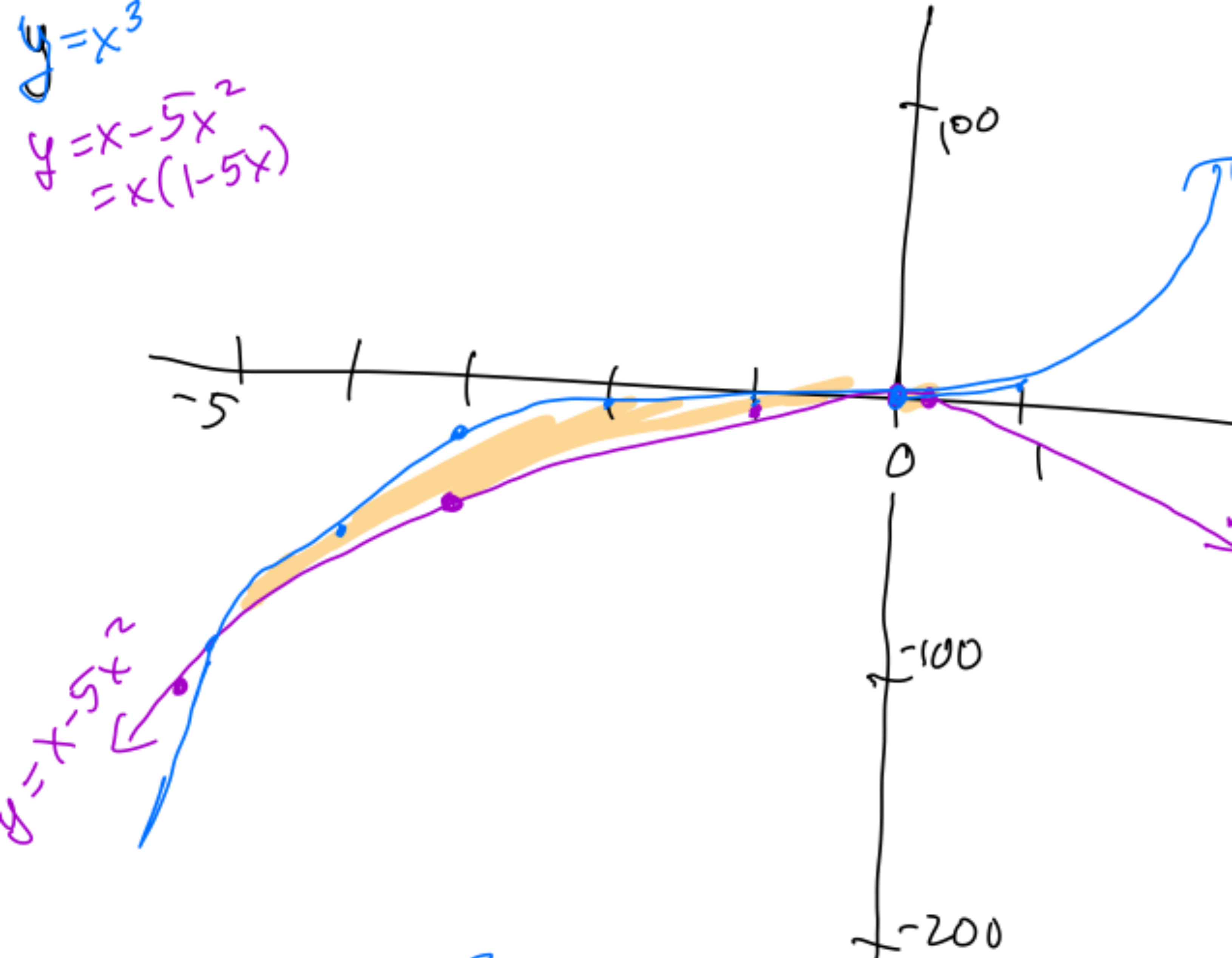
Aaaaaah....

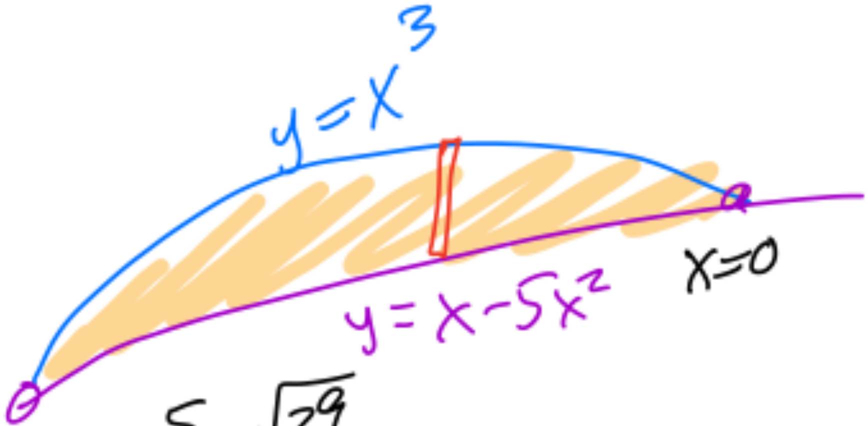
$x = \dfrac{-5 \pm \sqrt{25+4}}{2}$

$x = \dfrac{-5 \pm \sqrt{29}}{2}$

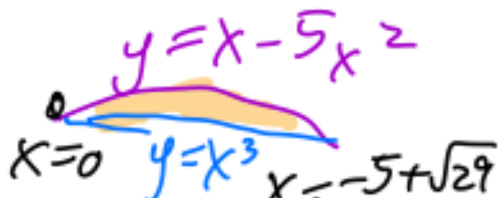
$y = x^3$

$y = x - 5x^2$
$= x(1-5x)$

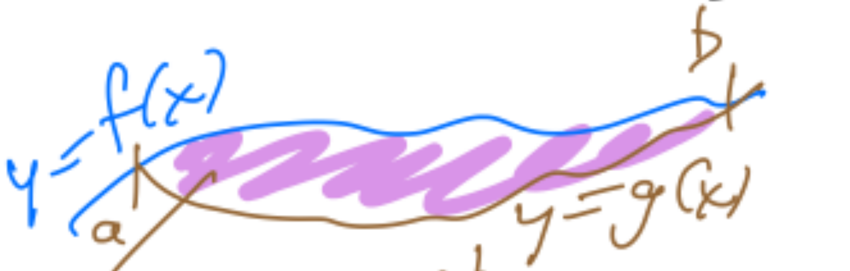




$x = \frac{-5-\sqrt{29}}{2}$



$x = \frac{-5+\sqrt{29}}{2}$



$\text{Area} = \int_a^b [f(x) - g(x)]\,dx$

Area between the two curves

$$= \int_{\frac{-5-\sqrt{29}}{2}}^{0} \Big[\underbrace{(x^3) - (x - 5x^2)}_{x^3 - x + 5x^2} \Big] dx$$

$$+ \int_{0}^{\frac{-5+\sqrt{29}}{2}} \Big[\underbrace{(x - 5x^2) - x^3}_{x - 5x^2 - x^3} \Big] dx$$

$$= \left(\frac{x^4}{4} - \frac{x^2}{2} + \frac{5}{3}x^3 \right) \Bigg|_{\frac{-5-\sqrt{29}}{2}}^{0}$$

$$+ \left(-\frac{x^4}{4} + \frac{x^2}{2} - \frac{5}{3}x^3 \right) \Bigg|_{0}^{\frac{-5+\sqrt{29}}{2}}$$

$$= 0 - \left(\frac{\left(\frac{-5-\sqrt{29}}{2}\right)^4}{4} - \frac{\left(\frac{-5-\sqrt{29}}{2}\right)^2}{2} + \frac{5}{3}\left(\frac{-5-\sqrt{29}}{2}\right)^3 \right)$$

$$+ -\frac{\left(\frac{-5+\sqrt{29}}{2}\right)^4}{4} + \frac{\left(\frac{-5+\sqrt{29}}{2}\right)^2}{2} - \frac{5}{3}\left(\frac{-5+\sqrt{29}}{2}\right)^3$$

$$- 0$$

Area!